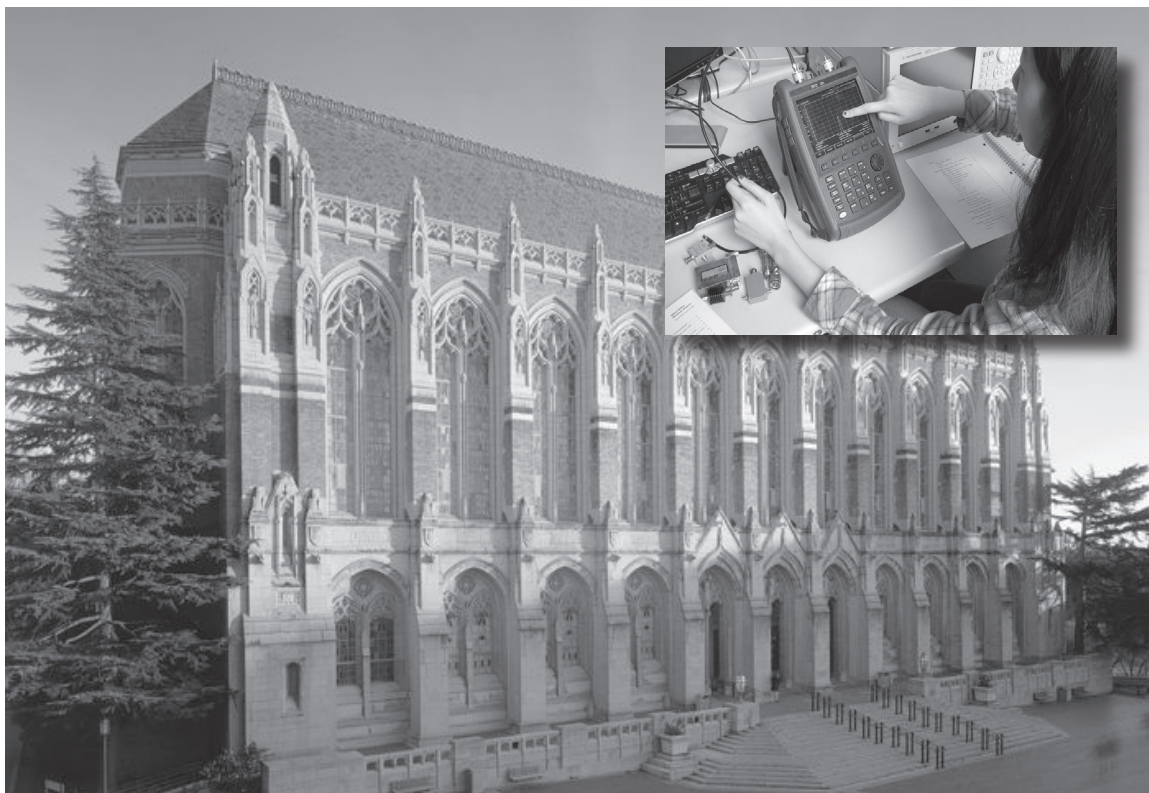


Keysight Technologies

Two-port Measurements and S-Parameters

University Engineering Lab Series - Lab 5

Application Note





Introduction

Network analyzers are the fundamental instrument for characterization of the devices and components used in RF and microwave systems. Network analyzers were briefly introduced in the previous laboratory for their use in impedance matching; they now will be examined in more detail within the context of component measurement. Transfer functions are more naturally expressed in the Laplace or frequency domain, and swept frequency instruments, like network analyzers, can provide a direct measurement of these transfer functions by sweeping a signal source and a tuned receiver across a range of frequencies. Two-port measurements provide explicit characterization of linear system blocks with a single input and a single output, by far the most common type of element. A two-port network could be as simple as a section of cable, or as complex as a complete transmitter-receiver link. In its simplest format, a reference excitation is applied to one port, and the response from the other port is recorded as a function of the sweep frequency. More complex elements with more than two ports can be characterized by extension, and the measurement techniques extend easily by simple multiplexing of the stimulus and response ports. Scattering parameters, or S-parameters for short, are the working language of network analyzers (NA). They provide a complete description of any linear, time-invariant element which then fully represents the behavior of that element within any system that it may be connected to. Scattering parameters involve phase information, and they are thus complex phasor quantities, and they are also frequency dependent. Once known, they can be transformed into other network parameters for circuit design, optimization, or tuning.

A quick overview

An electrical port is a pair of terminals across which a unique voltage and current can be defined, and through which an electrical signal or power can flow. A two-terminal element such as a resistor, capacitor, or inductor is a one-port device. The characteristics of that element define a relationship between the port voltage and the port current, for example $V = IR$, or $V = L di/dt$. One of these variables, say voltage, can be considered as a stimulus to the element, and the other variable, say current, can be considered as the response. The characteristics of the two-terminal element thus describe an electrical stimulus-response relationship.

A two-port device or network is the simplest electrical element with a unique input and output. A generic two-port network is illustrated in figure 1 below where the port variables of voltage and current are denoted in their more conventional circuit theory manner.

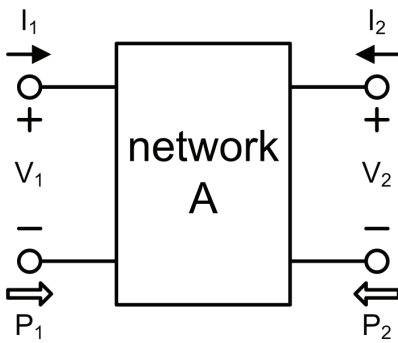


Figure 1. A two-port network with conventional circuit variable definitions

Because of the larger number of variables (now four), a two-port network requires a more elaborate method of characterization, and several matrix approaches exist for organizing this process. One such method is the impedance matrix, or Z-matrix. The Z-matrix representation of a two-port network can be thought of as having the two port currents, I_1 and I_2 , be the independent stimuli, and the two port voltages, V_1 and V_2 , be the responses that arise from those stimuli. Written out, the impedance matrix relationship is

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Reversing the roles of the stimuli and responses gives an alternative representation, the admittance matrix, or Y-matrix,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Simple matrix algebra shows that the impedance and admittance matrices are inverses of each other, $Y = Z^{-1}$. Numerous other representations are possible, each with different choices for the stimuli and responses. The more common ones are the G-, H-, and T- or ABCD-matrices, which are developed in other courses on linear network theory.

When dealing with high frequency systems where the propagation of signals is better described by travelling waves, the choice of stimuli and responses needs to be modified. The stimuli are best described by waves that are incident upon the ports, and the responses are best described by waves that are reflected back from those same ports. Figure 2 illustrates the same two-port network as in figure 1, but with the port variables now labeled as incident (in-going, +) and reflected (out-going, -) voltage waves.

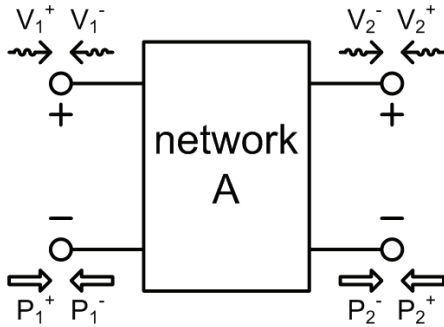


Figure 2. A two-port network with incident and reflected field variable definitions

The scattering matrix or S-matrix is the representation which describes this choice,

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

While providing a more intuitive description in terms of propagating waves, the scattering parameters also match precisely to the manner in which they are measured in the laboratory with network analyzers.

For a single port, the scattering matrix becomes simply,

$$\begin{bmatrix} V_1^- \end{bmatrix} = \begin{bmatrix} S_{11} \end{bmatrix} \begin{bmatrix} V_1^+ \end{bmatrix}, \text{ or } V_1^- = \Gamma V_1^+$$

where $\Gamma = S_{11}$ is the reflection coefficient that has been described previously and measured directly with the FieldFox NA application.

The S-matrix representation of the two-port network still involves four port variables: two incident stimuli and two reflected responses, each physically measured in volts as shown in figure 2. But what happened to the port currents? Propagating electromagnetic waves maintain a relationship between their electric and magnetic fields, and that ratio is the characteristic impedance Z_0 which is determined by the media of propagation. As guided waves propagating along transmission lines, the ratio of electric to magnetic field strength manifests itself as a ratio of voltage to current with exactly the same value, the characteristic impedance Z_0 . Each of the propagating voltage waves has associated with it a propagating current wave whose amplitude is simply the voltage divided by Z_0 , e.g. $I_{1+} = V_{1+} / Z_0$. In other words, the currents have become implicit, since they are each the corresponding voltages divided by the same Z_0 factor, and therefore do not provide any new information. Dividing the scattering matrix equation by Z_0 gives the corresponding relationship for the currents,

$$\begin{bmatrix} I_1^- \\ I_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} I_1^+ \\ I_2^+ \end{bmatrix}$$

It is possible to convert between the circuit variables of figure 1 and the field variables of figure 2 by noting that the circuit variables are the superposition of the forward and reverse propagating wave amplitudes. For the two port voltages,

$$V_1 = V_1^+ + V_1^- , \text{ and } V_2 = V_2^+ + V_2^-$$

For the two port currents,

$$I_1 = I_1^+ - I_1^- , \text{ and } I_2 = I_2^+ - I_2^-$$

where the negative sign accounts for the reflected current flowing in the opposite direction to the incident.

Power flow into the ports can also be computed using either circuit or field variables. Using the circuit variables of figure 1,

$$P_1 = V_1 I_1^* , \text{ and } P_2 = V_2 I_2^*$$

where the complex conjugate is needed because all of the port variables are phasors.

Using the field variables of figure 2,

$$P_1 = P_1^+ - P_1^- = V_1^+ I_1^{+*} - V_1^- I_1^{-*} , \text{ and } P_2 = P_2^+ - P_2^- = V_2^+ I_2^{+*} - V_2^- I_2^{-*}$$

Conventionally, current flows and power flows are defined as flowing into the port of the network under consideration.

Now consider the simple example of a lossless transmission line of length l and of characteristic impedance Z_0 with a propagation constant of β . To compute the Z-matrix parameters, each of the termination conditions must be properly applied. For example, the Z_{11} parameter is obtained as the ratio of V_1 to I_1 , the input impedance, under the condition that the opposite port is open-circuited to make $I_2 = 0$:

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0}$$

An open-circuit condition on port-2 will create a reflection coefficient of $\Gamma = +1$ at that port, and then transforming that back to port -1 will create an input impedance of

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = -jZ_0 \cot \beta l$$

using $Z_L \rightarrow \infty$. The rest of the Z-matrix can be computed in a similar manner yielding

$$[Z] = \frac{-jZ_0}{\sin \beta l} \begin{bmatrix} \cos \beta l & 1 \\ 1 & \cos \beta l \end{bmatrix}$$

The same lossless transmission line would have a T-matrix (also known as an ABCD matrix) of

$$[T] = \begin{bmatrix} \cos \beta l & jZ_0 \sin \beta l \\ jY_0 \sin \beta l & \cos \beta l \end{bmatrix}$$

The scattering matrix for the lossless transmission line ends up being particularly simple. To compute the S_{11} element, the terminating condition on the opposite port is for no incident signal, $V_2^+ = 0$. This would be achieved by terminating port-2 with a matched load of $Z_L = Z_0$. That way, any signal being emitted from port-2 would not create any reflection going back into port-2. With port-2 terminated in a matched load, the input impedance to port-1 is just the characteristic impedance of Z_0 , regardless of the length of the line. This creates no reflections and thus $S_{11} = 0$. By similar reasoning and symmetry, $S_{22} = 0$ also.

The off-diagonal elements of the scattering matrix represent the transmission from one port to another. A voltage wave entering port-1 and leaving port-2 will simply be delayed by a phase factor that is proportional to the length of the transmission line. This phase delay factor of $e^{-j\beta l}$ is the scattering parameter for transmission in either direction through the line, S_{12} or S_{21} . The scattering matrix for the lossless transmission line is therefore

$$[S] = \begin{bmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{bmatrix}$$

The simplicity of the S-matrix over the Z- or T-matrices for the lossless transmission line illustrates the great utility of using scattering parameters when fields and waves are the more apt description of a system.

As another example, consider the very common situation of a transmission line attenuator. This is a two-port network whose purpose is to attenuate the signal passing through by a fixed amount without introducing any reflections for signal propagation in either direction. These are most commonly created as either Π - or T-sections, the T-section illustrated in figure 3.

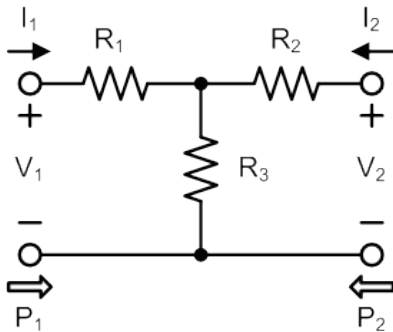


Figure 3. A T-section transmission line attenuator

First, it should be noted that an attenuator cannot be simply a single resistor inserted into the transmission line. This would cause the input and output impedances to no longer be Z_0 , and this would introduce unwanted signal reflections. The reason for needing 3 resistors to build the attenuator is that there are 3 separate requirements which must be satisfied. The voltage division factor from one port to the other must produce the intended attenuation factor, and the input impedance looking into each port must be the line impedance when the opposite port is properly terminated in $Z_L = Z_0$.

As an example, consider the design of a -3 dB attenuator for a $Z_0 = 50 \Omega$ transmission line. This device needs to reduce the transmitted power by a factor of 2, $10^{0.3} = 2$, and thus the voltage division from input to output must be a factor of $1/\sqrt{2} = 0.707$. When port-2 is terminated in a matched load of $Z_L = Z_0 = 50 \Omega$, voltage division from each of the two branch loops the circuit of figure 3 requires that

$$\frac{V_2}{V_1} = \frac{1}{\sqrt{2}} = \frac{(Z_0 + R_2) \parallel R_3}{(Z_0 + R_2) \parallel R_3 + R_1} \cdot \frac{Z_0}{R_2 + Z_0}$$

With a matched load on port-2, the input impedance must equal the line impedance,

$$Z_{in} = (Z_0 + R_2) \parallel R_3 + R_1 = Z_0$$

And with a matched load on port-1, the output impedance must also equal the line impedance,

$$Z_{out} = (Z_0 + R_1) \parallel R_3 + R_2 = Z_0$$

The above three equations need to be solved for the three unknowns of R_1 , R_2 , and R_3 . By symmetry, the last two equations result in $R_1 = R_2$. The denominator of the first quotient in the first equation can be recognized as $Z_{in} = Z_0$, which simplifies this to

$$\frac{1}{\sqrt{2}} = \frac{R_3}{Z_0 + R_2 + R_3}$$

Combining this with either of the impedance requirements produces

$$R_1 = R_2 = Z_0 \frac{\sqrt{2} - 1}{\sqrt{2} + 1} = 8.58 \Omega$$

Substituting back and solving for the third resistor gives

$$R_3 = \frac{Z_0 + R_1}{\sqrt{2} - 1} = 141.42 \Omega$$

It can be verified that the Z-matrix for this attenuator is

$$[Z] = \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix}$$

and that the transmission or T-matrix is

$$[T] = \frac{1}{2\sqrt{2}} \begin{bmatrix} 3 & Z_0 \\ Y_0 & 3 \end{bmatrix}$$

Because of the lack of any reflections by design, the scattering matrix is particularly simple,

$$[S] = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

The above S-parameters for a transmission line and an attenuator will be verified through network analyzer measurements later in this laboratory.

The vector network analyzer (VNA) is the principal tool used to measure the transfer functions of network components, and its versatility comes from its ability to handle multiple ports in a consistent and standardized manner. The native output from any VNA measurement is a scattering parameter, and this is the reason why so much of RF and microwave design is carried out in this representation. The output measurements from a VNA are fundamentally ratios of output to input, and while this naturally conveys the notion of a transfer function, from an instrumentation point of view, the measurement being a dimensionless ratio allows the VNA to produce accurate measurements over an extremely wide frequency range and also over an extremely wide dynamic range of signal amplitudes. One of the working principles of VNA design is that the ratios of two identical receivers will cancel out any particular transfer function of the receivers themselves, leaving a ratio of output to input which is undistorted by the particular characteristics of each receiver channel. In this manner, the bandwidth of the network analyzer can be actually greater and the response flatter than the bandwidth or flatness of its individual receivers.

The architecture of a generic vector network analyzer is shown below in figure 4.

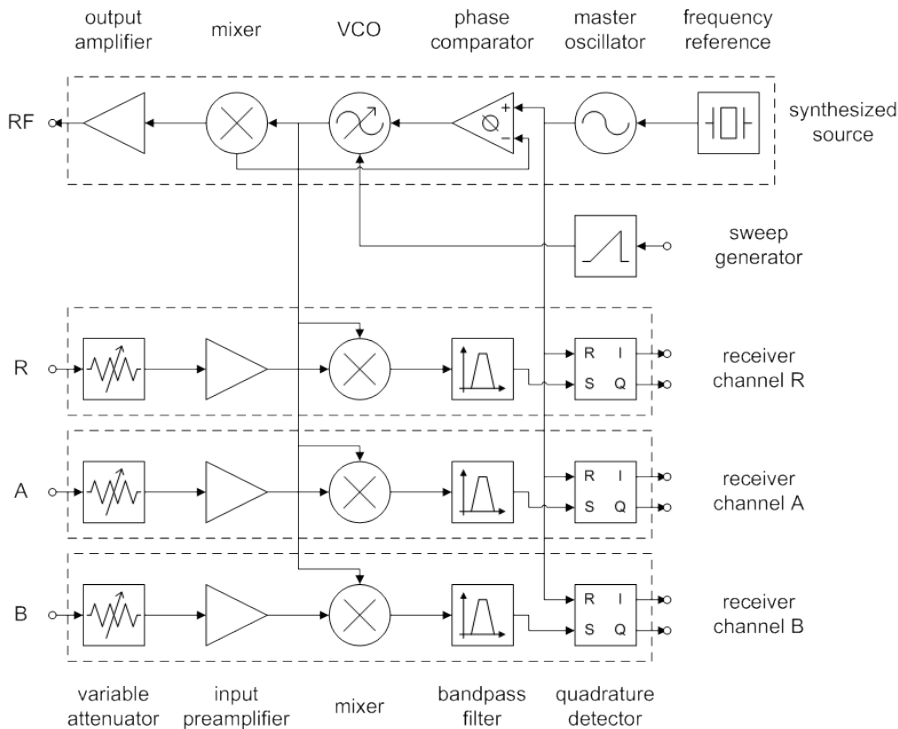


Figure 4. Vector Network Analyzer (VNA) architecture

As shown above the heart of the network analyzer is a synthesized signal source which creates the RF excitation and a set of identical tuned receiver channels which track the output from the synthesized source. In the above case there are three identical receiver channels, R, A, and B, but other network analyzer architectures may have more than these. The FieldFox analyzer used in this lab has four receivers. The synthesized signal source starts with a master oscillator with a stabilized frequency reference. This is most commonly a quartz crystal oscillator which is placed inside a temperature controlled oven to stabilize it against thermal variations. Such oscillators are usually accurate to at least 5-10 ppm once the oscillator oven has come up to temperature and had time to stabilize. The rest of the synthesized signal source is a subsystem known as a phase-locked loop (PLL). This is a feedback control system which creates new frequencies from a given reference by matching their phases. The critical components are a voltage-controlled oscillator (VCO), a phase comparator, either a frequency multiplier or divider, depending upon whether the output frequency needs to be greater than or less than the input reference frequency, and a loop filter for proper tracking and capture behavior. The loop filter is not shown in the above figure for simplicity. Phase-locked loops are extremely useful and flexible circuits which allow precise digital control over high frequency signal sources. The above synthesized signal source allows the network analyzer to output an RF excitation signal of a precisely selected frequency, and furthermore to sweep that signal under accurate digital control.

As shown, the receiver side of the network analyzer consists of several identical tuned receivers. Each measurement made with a network analyzer is fundamentally a ratio of two receiver channels. By making the receiver channels identical, their own transfer functions cancel in the measurement ratios. Each receiver consists of a programmable attenuator, an input preamplifier, a mixer to move the input RF signal down to a more comfortable intermediate frequency IF signal, a band pass filter to select just the IF portion from the mixer, and a quadrature detector to measure the IF signal as a full vector quantity. The receiver attenuator, preamplifier, mixer, and band pass filter operate in exactly the same manner as the receiver that was previously described for a spectrum analyzer. The big difference lies in the type of detector.

A spectrum analyzer is only concerned with the scalar amplitude of the signal, so a fairly simple diode envelope detector is sufficient for that purpose. The 'vector' in vector network analyzer comes from the fact that each receiver channel has a detector which is phase sensitive. This allows the full vector form of the received signal to be measured, both amplitude and phase. Instead of trying to measure a phase angle directly, the quadrature detector measures two orthogonal components of the signal, the in-phase I signal and the quadrature Q signal. The I and Q are 90° apart in phasor space, and are physically implemented by mixing the input signal with a sine and a cosine version of the reference frequency. The I and Q thus produce the real and imaginary parts of the received signal, and from these the amplitude and phase angle can be computed through a rectangular to polar conversion.

It is worth mentioning that there exist other strategies for obtaining this vector signal information and the quadrature detector scheme is only one of several commonly used. A very common alternative is to use a high speed analog sampler which has the effect of producing a comb filter. This is a very cost effective way of producing a network analyzer receiver when a low jitter sampling gate is available for the frequency range of interest.

The VNA architecture shown in figure 4 is still some distance away from being able to make an RF scattering parameter measurement. In order to connect a component to the VNA and have all of the various interconnection components not introduce their own transfer function distortions, a specially designed test set is required. A simple test set and its connection to the device under test (DUT) is shown in figure 5. This configuration is often known as a transmission/reflection or T/R test set. The RF, R, A, and B connections tie directly back to the network analyzer connections of the same name in figure 4. The RF excitation signal from the network analyzer synthesized source is first fed into a power splitter which sends half of the RF power back to the Reference or R receiver channel. The remaining half is fed into a directional coupler which sends the signal on to the port 1 connection where port 1 of the DUT is attached. The excitation signal passes into the DUT through port 1, and the signal that is transmitted through the DUT emerges at port 2. The test set simply passes this transmitted signal back to the network analyzer receiver channel B. The transmission of the DUT is then measured by the ratio of the B channel receiver to the R reference channel receiver, so $\text{transmission} = B/R$.

The directional coupler is a particularly interesting component all to its own, and it will be studied in more detail in another lab. For the present purposes, the directional coupler works as a signal sampler which peels off a small fraction of the signal passing through it, but only that part which is propagating in a specific direction. The coupling of a directional coupler is typically about 10 to 20 dB, meaning that the sampled signal output power is only 1/10 or 1/100 of the signal passing straight through. The directional coupler is a purely passive device and it maintains an accurate coupling factor over its entire frequency range of operation.

Referring to figure 5, the forward propagating signal from the power splitter to port 1 will couple a sampled signal that emerges from the lower right port of the directional coupler. This sampled signal is not needed, since the power splitter already provides an input reference level back through the R-channel. Since this sampled signal is not needed, this port of the directional coupler is terminated in a matched load to eliminate any unwanted signal reflections. Similarly, the signal which is reflected back from port 1 of the DUT propagates the other direction through the directional coupler, and its sampled signal emerges from the lower left port of the directional coupler which is fed back to the A-channel receiver of the network analyzer. The reflected signal from the DUT is thus measured as the ratio of the A-channel to the R-channel, so $\text{reflection} = A/R$, after correcting for the coupling factor of the directional coupler which is a known fixed quantity for a given test set.

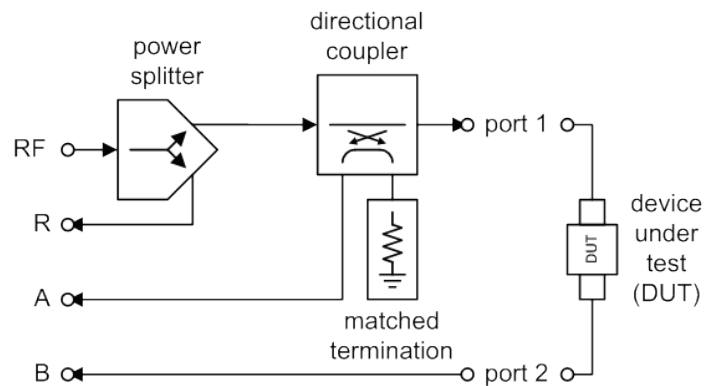


Figure 5. A simple T/R test set and DUT connections

In this manner, the T/R test set allows two of the four scattering parameters to be measured: the reflection coefficient from port 1, S_{11} , and the transmission from port 1 to port 2, S_{21} . The remaining two scattering parameters S_{12} and S_{22} could be measured using the same T/R test set but by simply reversing the DUT between ports 1 and 2.

The T/R test set shown above is adequate to measure any scattering parameter of any network, but in practice the process will involve a lot of assembling and disassembling of connectors, which will make it tedious and time consuming. A better solution is to do this switching electronically and automatically. The full S-parameter test set of figure 6 adds a second directional coupler so that the signals emerging from both port 1 and port 2 can be measured by channels A and B, respectively. It also adds a double-pole, double-throw (DPDT) relay which is used to direct the RF excitation into either port 1 or port 2, while keeping the other port properly impedance matched with a load termination.

With the DPDT switch in the position shown in figure 6, the RF excitation is directed to port 1, and the signals returning from port 1 and port 2 are measured by receiver channels A and B. This allows the measurement of S_{11} and S_{21} . Changing the DPDT switch to the other position then directs the RF excitation to port 2, and the signals returning from port 1 and port 2 are again measured by receiver channels A and B. This second position of the DPDT relay allows the measurement of S_{12} and S_{22} . The relay is usually automatically switched by the instrument microcontroller depending upon which S-parameter measurement is desired.

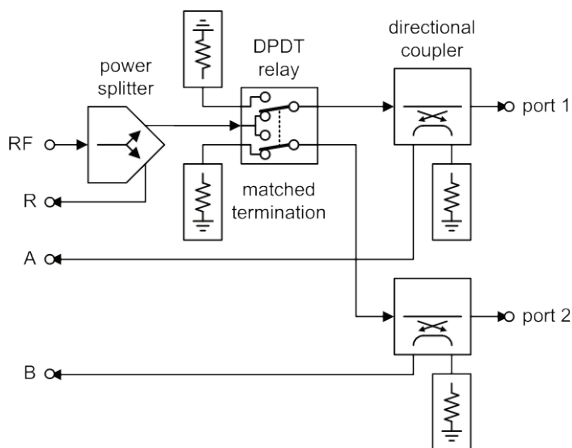


Figure 6. A full two-port S-parameter test set

This type of test set and three-receiver architecture works well for many applications, but it relies upon the DPDT relay being nearly ideal in its RF characteristics. A more accurate, but costly, approach is to interchange the order of the DPDT relay and the power splitter, thereby having a power splitter for each port which produces reference signals R1 and R2 for the two ports. This type of network analyzer uses four receivers to produce the measurements A/R1 and B/R2, which is more accurate because the characteristics of the DPDT relay are now out of the picture. Most of today's higher-quality VNAs use four receiver architectures.

Some network analyzers take this a step further and offer the ability to take S-parameter measurements from more than just two ports. Four-port test sets are commonly used to measure devices with more than two ports such as mixers, directional couplers, power splitters, and power combiners. Their advantage lies in the ability to connect the DUT once and then let the instrument take care of routing the source and receivers to the proper pairs of ports. An example is the Keysight E5071C-460, which is a 4-port 6 GHz model.

Older generation vector network analyzers often had the test set as a separate chassis. Typical examples of those would be the HP 8754A network analyzer and the HP 8748A S-parameter test set, or the HP 8753C network analyzer and the HP 85047A S-parameter test set, shown in figure 7. Today, most VNAs have the test set integrated into the instrument. The Keysight E5071C series and N9914A FieldFox each have fully integrated test sets.



Figure 7. An HP-8753C 6 GHz network analyzer on top with its matching HP-85047A S-parameter test set below. The RF, R, A, and B signal connections between the two instruments are clearly visible.

Some practice two-port measurements

The Keysight N9914A FieldFox will now be used to make some typical S-parameter measurements on a few simple components whose response should be predictable and understood. The simplest of these is a short section of coaxial cable.

Set up the Keysight N9914A FieldFox to operate on the external power supply. Start up the FieldFox and after it has loaded and launched the default CAT application, press the Mode button and then the NA soft key to load and launch the Network Analyzer application. If they are not already present, install a Type-N to SMA adapter on port 1 and port 2 of the FieldFox. The NA application starts up showing the S_{11} parameter over the full frequency range of 30 kHz to 6.5 GHz and presents this in a log-magnitude format. With nothing attached to port 1, this should be a nearly flat yellow line across the screen centered over 0 dB, indicating nearly 100% reflection from port 1.

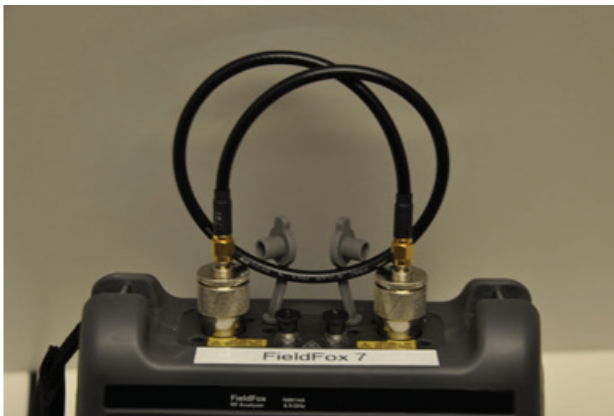


Figure 8. Measurement of a coaxial cable using the FieldFox

Connect one end of an 18-inch long coaxial cable with SMA connectors to port 1, and notice that there is a little drop in the S_{11} trace, but it still remains fairly close to the 0 dB level. Next, connect the remaining end to port 2 which will now terminate this end in the $50\ \Omega$ impedance of port 2, as shown in figure 8. The S_{11} trace should drop appreciably to at least the -10 dB level or lower. Port 2 is now absorbing most of the power that is being sent into the cable from port 1, and the reflection from port 1 is therefore much less. Ideally, this would be zero if all parts of the cable were perfectly matched. There are no perfect connectors, nor is any cable perfectly matched to the impedance of the test ports. These non-ideal features are the origin of structure shown in the S_{11} trace.

Next press the S_{21} soft key to display a measurement trace for the S_{21} scattering parameter. Here, the trace should look fairly flat and close to the 0 dB level, indicating that nearly all the power incident from port 1 is being transmitted through and received by port 2. The small drop at higher frequencies is due to cable ohmic and dielectric loss, as was investigated in an earlier laboratory. Press the S_{12} and S_{22} soft keys to display measurements of the remaining S-parameters. Because the cable is symmetrical, the S_{11} and S_{22} parameters should look similar, as should the S_{21} and S_{12} parameters.

It should always be borne in mind that each of the S-parameters are vector quantities with real and imaginary parts. The present log-magnitude display format only shows the magnitude. The FieldFox offers several other display formats which can be selected by pressing the Format soft key. Press the Format soft key and then the Smith soft key to change the display format over to a Smith chart. This is a polar plot of the S-parameter with Smith chart impedance coordinate circles overlaid on to the unit circle.

The same measurement trace can also be displayed with a conventional polar coordinate grid by pressing the Format, More, and Polar soft keys. Notice that the only difference between the Smith and Polar display formats is the coordinate grid. Cycle through the four different S-parameters to observe what each of these looks like on the Smith chart. The S_{11} and S_{22} parameters should be clustered close to the origin, since the reflection from a matched line should be small. The S_{21} and S_{12} parameters should follow the outer edge of the unit circle because the transmitted signal should be close in magnitude to what was incident.

Because of the very wide sweep range (30 kHz to 6.5 GHz) each Smith chart trace will involve many loops around the origin. Change the frequency sweep range to run from 50 to 150 MHz. To do this, press the Freq/Dist button, change the frequency range limits, and then press the Measure button to return to the selection of S-parameters to measure. This should reduce the measurement trace to only 2/3 of one loop around the origin, greatly simplifying the display.

Over this more limited frequency range, the magnitude of each of the S-parameters is relatively constant, but each traces circles around the origin, indicating a constantly changing phase. Pick off a specific point on the trace by using the Marker function. Press the Marker button and enter a desired frequency. To display the marker in magnitude-phase format, press the More, Marker Format, and Mag & Phase soft keys. Use the dial on the FieldFox to move the marker around the trace and verify that the phase angle decreases, i.e. the marker moves around the Smith chart in a counter-clockwise direction, as the frequency increases. This should be true for all four of the S-parameters. Phase delay is the negative of the phase angle, so a decreasing phase angle implies an increasing phase delay. As the frequency increases, the wavelength of the signal becomes shorter, and for a given length of cable, more wavelengths will fit into this length. Increasing frequency therefore creates a linearly increasing phase delay (a linearly decreasing phase angle) in a non-dispersive cable, that is, one for which all frequencies propagate at the same velocity.

The phase angle can be displayed more clearly by changing the display format. Press the Measure button, followed by the Format and Phase soft keys. The display now shows the phase angle of the selected S-parameter versus frequency. Notice that this is linearly decreasing for all four of the S-parameters. The S_{11} and S_{22} parameters may show a vertical jump in the display; this is simply a roll-over from -180° to $+180^\circ$. If you change the format to unwrapped-phase, you can clearly see the linearly decreasing phase.

Set the format to Phase (not unwrapped). Use the marker function to find the frequency at which the S_{21} parameter has a phase angle of exactly -90° ; this should be close to 100 MHz. This is an important data point, because at this frequency the coaxial cable is exactly $1/4$ of a wavelength long. A quarter-wave section of transmission line has some very unique and useful properties, the most fundamental being the transformation of the load impedance by a reciprocal function.

Quarter wavelength points provide a useful means for measuring the time delay through a cable or network. The quarter wavelength -90° phase angle means that $1/4$ of a cycle has elapsed in getting the signal from port 1 to port 2. In the present case, this occurred at a frequency of about 100 MHz. The time delay through the cable is therefore about $\Delta t = 1/4f = 2.50$ ns. Compute the time delay using the more accurately measured quarter wavelength frequency and save this value; it will be needed later. The use of a quarter wavelength point simply makes the computation easier and it simplifies picking the frequency point by providing a known target value of -90° phase angle. In principle this method could be performed at any frequency, but the fractional part of a cycle would need to be known. More generally,

$$\Delta t = \frac{-\angle S_{21}}{360^\circ f}$$

This particular method of computing an accurate time delay has the advantage of not needing the cable length. This makes it less error prone than measuring the cable length and velocity factor. It also leverages the feature that network analyzer phase measurements are extremely accurate.

Complete the cable measurement by measuring and recording the four S-parameters at a frequency of 100 MHz. Use the marker function to obtain the magnitude and phase of each S-parameter.



Figure 9. Measurement of a coaxial attenuator using the FieldFox

Next, the S-parameters for an attenuator will be measured. Disconnect the SMA cable from port 2 of the FieldFox and insert a Mini Circuits VAT-10+ 10 dB attenuator in between the cable and port 2, as shown in figure 9. Keeping the frequency sweep range of 50 to 150 MHz, first measure the S_{11} parameter on a log-magnitude display. The first thing that should be noticed is that the S_{11} parameter is much closer to ideal than it was for the cable by itself. This results from the attenuator absorbing power that passes either way through it, greatly reducing any back reflections. The S_{21} parameter should likewise look quite ideal, a nearly perfect -10 dB straight line on a log-magnitude display. Likewise, the S_{12} parameter should look the same. Compare the S_{11} and S_{22} parameters. Unlike the case for the coaxial cable by itself, these two measurements are now different. Provide an explanation for why they are different. As a hint, the order of the components makes a difference in a cascaded connection.

Next, display the S_{21} parameter using a phase display format. Note that the phase angle is once more linearly decreasing with frequency. Use the marker function to find the frequency at which the phase is -90° . This is the quarter wavelength frequency for the cable and attenuator combination. This should be a value closer to 90 MHz, slightly lower in frequency than that of the cable by itself to account for the additional delay that is introduced by the attenuator. If the quarter wavelength frequency were 90 MHz, that would give a total time delay of $\Delta t = 1/4f = 2.78$ ns. The additional time delay of 2.78 ns $- 2.50$ ns = 0.28 ns is the time delay for signals passing through the attenuator. Using the more accurate measured frequencies, compute the time delay for the attenuator.

This type of time differencing measurement ends up being quite accurate because the Mini-Circuits VAT-10+ attenuator is an insertable element. Because it has a male and female SMA connector, the Mini-Circuits attenuator can be inserted into a section of line with no other components needed. If even a small coaxial adapter were used, this would introduce some additional time and phase delay, and possibly some attenuation as well. Using insertable elements eliminates this source of error. For this reason, it is often common to measure elements as if they were insertable by adding coaxial adapters to make them have both male and female ends. It is important to realize that the resulting measurements then pertain to the element with these additional adapters attached.

Complete this measurement by recording the four S-parameters for the cable and attenuator combination at a frequency of 100 MHz. Use the marker function to pick out these values in Mag & Phase format. Record them for use later.

Finally, to see how well this attenuator performs at higher frequencies, increase the frequency sweep range to the full span of the FieldFox, 30 kHz to 6.5 GHz. Examine the scattering parameters on a log-magnitude display format to see how well the attenuator performs across the whole band. Comment on the behavior by noting any frequency points at which the S-parameters become significantly poorer, as compared to what an ideal attenuator should produce.

Some further analysis

It should be clear from the previous measurements that the observed S-parameters for a given DUT depend upon the lengths of the coaxial cables that are used to connect the DUT to the test ports of the network analyzer. Yet the objective of these measurements is to characterize just the DUT in the absence of any test port cables. How are the obscuring characteristics of the test cables removed from the measurement? There are several different approaches.

The effective points at which the network analyzer considers the external device to start are known as the test planes or reference planes. A test plane forms a dividing line between the instrument and what the instrument will be measuring. When first turned on, the default test planes for the FieldFox are the two Type-N connectors on the top of the instrument. The two Type-N to SMA coaxial adapters, any coaxial test cables, and the DUT itself all collectively become the external system that the FieldFox is taking measurements on. Any coaxial adapters and test cables introduce phase delay and possibly attenuation and reflections into the overall measurement. Unlike in low-frequency electronics, the length of the interconnecting cable affects matters significantly.

Simple phase delays associated with a length of line l can be taken into account by noticing that their effect is to multiply the forward and reverse travelling waves by a factor of

$$e^{-j\beta l} = e^{-j2\pi l/\lambda} = e^{-j2\pi f l/v} = e^{-j\omega l/v} = e^{-j\omega \Delta t}$$

Most network analyzers can mathematically remove these phase delays by a procedure known as port extension. Extending the port means moving the test plane outward from the instrument to a new location that is specified by a propagation time delay Δt . If the test planes for both ports are extended outward to the connectors of the DUT, then the test cables and any associated adapters will then be treated as part of the instrument, at least in terms of phase delay. However, this process does not remove the effects of attenuation or reflections introduced by the cables and adapters. But for short length, high quality test cables, port extension is a very fast method to get closer to the real S-parameters of the DUT.

Port extension on the FieldFox is easy to accomplish. Remove the Mini-Circuits attenuator and connect the SMA coaxial cable between port 1 and port 2 of the FieldFox. Set the frequency sweep for a range of 50 to 150 MHz. Measure the S21 parameter and find the quarter wavelength frequency for the cable by using the marker function and a phase display format. This was done previously, but it can be checked again here. Compute the time delay of the cable from this quarter wavelength frequency measurement; this should produce a value near to 2.50 ns which represents the phase delay of the cable and both coaxial adapters. This value can be double-checked by using the marker to delay function of the network analyzer. Place a marker in the center of the trace and use the Mkr→Delay function. Then look under Scale, More, and note the reported delay. It should also be approximately 2.50 ns. Change the electrical delay back to zero to return to the measurement. This overall phase delay needs to be distributed between port 1 and port 2, but the distribution will depend upon where the DUT will be placed. If the DUT, the Mini-Circuits attenuator, will be inserted between the coaxial cable and the coaxial adapter on port 2, then most of the 2.50 ns delay will need to be applied to port 1 and only a very small amount to port 2.

To determine how to divide the 2.50 ns overall phase delay between port 1 and port 2, the phase delay from port 1 to the end of the coaxial cable needs to be measured. Presumably, this will be just a little bit shorter than 2.50 ns.

Disconnect the coaxial cable from port 2 so that the open circuit end of the cable produces a hard reflection back into port 1. Measure the S_{11} parameter and find the frequency at which the phase is 180° . Use the marker function and display the S_{11} parameter in phase format. The frequency for 180° phase shift will be easy to spot. It will occur where the trace makes an abrupt vertical jump from -180° to $+180^\circ$. Compute the phase delay for this situation, noting that the round trip involved in the S_{11} reflection will correspond to a one-way propagation delay of -90° . Since the frequency at which this occurs will be a little bit higher than the quarter wavelength frequency for the S_{21} measurement, this will produce a propagation delay time which is a little bit shorter than the 2.50 ns measured for S_{21} . Suppose that the 180° phase shift frequency for S_{11} was measured to be 104 MHz. That would give a propagation delay of 2.40 ns for the coaxial adapter on port 1 plus the coaxial cable. This would be the port extension time delay for port 1. Suppose that the quarter wavelength frequency for the S_{21} measurement was 100 MHz. This would give a propagation delay of 2.50 ns for the coaxial adapter on port 1, the cable, and the coaxial adapter on port 2 (all three parts). The difference between these, 0.10 ns, is the port extension time delay for port 2, and represents the delay for just the coaxial adapter.

To set these port extension delay times, press the Meas Setup button and then the Port Extensions soft key. First press the Port Extensions ON soft key to enable the port extensions. Press the Port1 Extension soft key and enter the 2.40 ns value. Press the Port2 Extension soft key and enter the 0.10 ns value. Press the Measure button to return to the S-parameter measurement selection menu. Note that the 2.40 ns and 0.10 ns values are only illustrative. Use the more accurate values that were computed based upon the measurements made on the specific cable and adapters that are being used.

Verify that these port extension delay times have properly nulled out the time delays of the cable and the coaxial adapters. With the coaxial cable still disconnected from port 2, but still connected to port 1, display the S_{11} measurement trace and verify that its phase is very close to 0° over the full frequency sweep. If displayed on the Smith chart, the S_{11} parameter should be very close to $1.00 / 0^\circ$. Connect the cable to port 2, display the S_{21} measurement trace, and verify that its phase is very close to 0° over the full frequency sweep. Likewise, displayed on the Smith chart, the S_{21} parameter should also be very close to $1.00 / 0^\circ$. The FieldFox and the cables and connectors are now all set up to measure the S-parameters of the attenuator without any interfering time delays from the test cable and connection hardware.

Insert the Mini-Circuits VAT-10+ attenuator between the coaxial test cable and the coaxial adapter on port 2. Use the marker function in Mag & Phase format to read off the four S-parameters at a frequency of 100 MHz. Record these four complex S-parameters which now represent a characterization of the Mini-Circuits attenuator. Compare these values to the ideal S-parameter matrix for a 10 dB attenuator.

There are several matrix methods for accomplishing the same function as the FieldFox port extensions. The more general problem is determining the S-parameter matrix for a cascade connection of two elements, each of whose individual S-parameter matrices are known. A two-port matrix particularly designed for dealing with cascaded elements is the transfer matrix, or T-matrix, also known as the ABCD-matrix. The transfer matrix specifically relates variables on one port to variables on the other,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The direction for I_2 is reversed from the normal convention of the current flowing into the port so that the port variables for one port can be used directly for the next element in the chain. Consider the cascade of two 2-ports, as shown in figure 10 below.

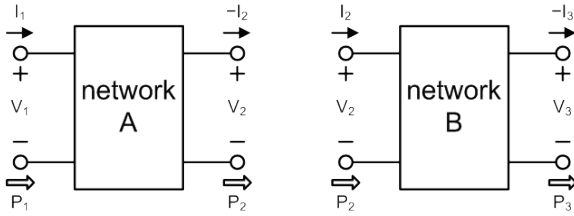


Figure 10. Cascade connection of two 2-port networks for T-matrices

Each of the two-ports can be described by their transfer matrices, T_A and T_B ,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = T_A \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}, \text{ and } \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = T_B \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix}$$

Simply multiplying the transfer matrices together gives the transfer matrix for the cascade combination of the elements,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = T_A T_B \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix}$$

As an example, multiplying the transfer matrices for a cable of length l and for a 10 dB attenuator would produce the resulting transfer matrix for the combination:

$$\begin{aligned} T_{\text{Cable}} T_{\text{Atten}} &= \begin{bmatrix} \cos \beta l & jZ_0 \sin \beta l \\ jY_0 \sin \beta l & \cos \beta l \end{bmatrix} \cdot \frac{\sqrt{10}}{2} \begin{bmatrix} 1.1 & 0.9Z_0 \\ 0.9Y_0 & 1.1 \end{bmatrix} \\ &= \frac{\sqrt{10}}{2} \begin{bmatrix} 1.1 \cos \beta l + j0.9 \sin \beta l & Z_0 0.9 \cos \beta l + jZ_0 1.1 \sin \beta l \\ Y_0 0.9 \cos \beta l + jY_0 1.1 \sin \beta l & 1.1 \cos \beta l + j0.9 \sin \beta l \end{bmatrix} \end{aligned}$$

This is a bit messy for hand computations, but perfectly efficient for numerical programs. This type of transfer matrix approach is well described in many textbooks on network theory.

Another matrix approach, although less often described in textbooks, proves to be more useful for S-parameter computations that need to be done at the network analyzer. Here, the S-parameter formulation is rearranged to express the variables at one port in terms of those at the other, similar to the transfer matrix concept, but instead using the forward and reverse travelling waves. For want of a better name, these shall be called the W-parameters. The advantage is that the W-parameters involve the same four variables as the S-parameters, $\{V_{1+}, V_{2+}, V_{1-}, V_{2-}\}$, and this results in usually simpler matrices which are easier to manipulate. Written out, the W-parameters are

$$\begin{bmatrix} V_1^+ \\ V_1^- \end{bmatrix} = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \begin{bmatrix} V_2^- \\ V_2^+ \end{bmatrix}$$

Notice that the port 1 and port 2 variables interchange their forward and reverse positions. This is so that the output of one port becomes the input to the next in a cascade connection. This is shown in figure 11 below. Here these definitions of the forward and reverse travelling waves give

$$\begin{bmatrix} V_2^- \\ V_2^+ \end{bmatrix} = \begin{bmatrix} V_3^+ \\ V_3^- \end{bmatrix}$$

In each ordered pair, the first variable is always propagating from left to right, and the second variable is always propagating from right to left. This is what allows multiplication of these matrices to properly represent cascading of the networks.

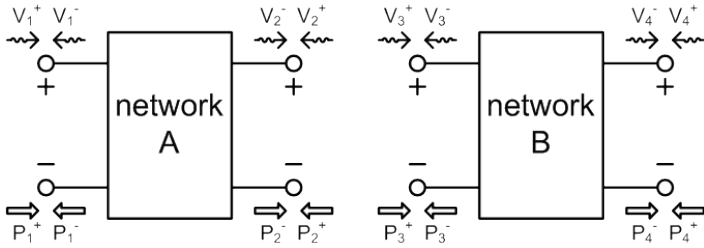


Figure 11. Cascade connection of two 2-port networks for W-matrices

The W-parameters can be determined from the S-parameters by simple algebra,

$$W_{11} = \frac{1}{S_{21}} \quad W_{12} = -\frac{S_{22}}{S_{21}}$$

$$W_{21} = \frac{S_{11}}{S_{21}} \quad W_{22} = \frac{S_{12}S_{21} - S_{11}S_{22}}{S_{21}}$$

The reverse transformation is

$$S_{11} = \frac{W_{21}}{W_{11}} \quad S_{12} = \frac{W_{11}W_{22} - W_{12}W_{21}}{W_{11}}$$

$$S_{21} = \frac{1}{W_{11}} \quad S_{22} = -\frac{W_{12}}{W_{11}}$$

The W-matrix for a cable of length l is

$$W_{Cable} = \begin{bmatrix} e^{+j\beta l} & 0 \\ 0 & e^{-j\beta l} \end{bmatrix}$$

and the W-matrix for a 10 dB attenuator is

$$W_{Atten} = \begin{bmatrix} \sqrt{10} & 0 \\ 0 & \frac{1}{\sqrt{10}} \end{bmatrix}$$

Direct multiplication of these matrices gives the W -matrix for the cascade connection,

$$W_{Cable}W_{Atten} = \begin{bmatrix} \sqrt{10}e^{+j\beta\ell} & 0 \\ 0 & \frac{1}{\sqrt{10}}e^{-j\beta\ell} \end{bmatrix}$$

Inserting a cable into the cascade chain is therefore equivalent to multiplying by the W -matrix for that cable. Removing the cable from the cascade chain is equivalent to multiplying by the inverse of the W -matrix,

$$W_{Cable}^{-1} = \begin{bmatrix} e^{-j\beta\ell} & 0 \\ 0 & e^{+j\beta\ell} \end{bmatrix}$$

The W -matrices for ideal cables and attenuators are particularly simple because the matrices are diagonal and have a determinant of unity, greatly simplifying matrix operations. It should be noted that matrix multiplication is non-commutative, so the order of the matrices being multiplied must match exactly to the order of the components in the cascade chain.

Test this procedure using the data that was collected for the cable and attenuator at 100 MHz. First, compute the W -matrix for the test cable from the S -parameters that were measured for the cable alone. Next, compute the W -matrices for the test cable and attenuator combined together, and then for the attenuator with the port extensions applied. Verify through complex matrix multiplication that

$$W_{Cable+Atten} = W_{Cable}W_{Atten}$$

From the above, it can be seen that ideal cables and attenuators can be easily taken into account. The port extension function of the FieldFox computes these in precisely this manner, simplifying the measurement process. However, port extension corrections only involve pure phase delays and do not include any effects of attenuation or reflections. To correct for possible imperfections in the test cables, adapters, and fixtures, a more comprehensive approach is needed. This is the topic of network analyzer calibration which will be taken up later. Calibration of a network analyzer allows the port extension process to include not only phase delays, but also any possible attenuation or reflections to be completely zeroed out. A properly calibrated network analyzer is capable of remarkable accuracy, but by the same token, a network analyzer whose calibration is unknown or not properly applied can lead to horrible mistakes in RF measurements.

Measurement assignment

Using the vector network analyzer techniques described above, characterize a Mini-Circuits SBP 70+, a 63 to 77 MHz band pass filter. This is a coaxial band pass filter with a center frequency of 70 MHz. Most of the action will occur in the range of 50 to 100 MHz, but feel free to extend the sweep range to examine other parts of the response.

Use the FieldFox to specifically determine (1) the insertion loss at 70 MHz, (2) the reflection coefficient magnitude at 70 MHz, (3) all four S-parameters at 70 MHz, (4) the rate of response roll-off outside of the pass-band, and (5) the number of poles in the filter which would produce this.

The Mini-Circuits SBP-70+ is an insertable coaxial component, so it can be measured using the same procedures as were developed for the insertable coaxial attenuator. Figure 12 shows how the band pass filter should be connected to the FieldFox for these measurements.

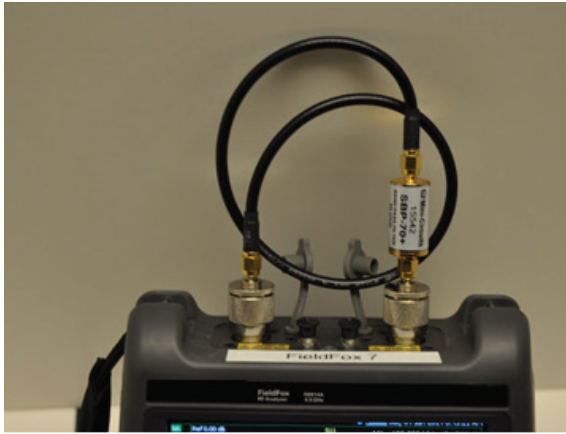


Figure 12. Measurement of a coaxial band pass filter using the FieldFox

The S_{21} log-magnitude response will also show a notch in the response above the pass band. Measure the frequency at where this occurs. What features of a passive network would be required to produce this?

Examine the S_{11} and S_{22} parameters on a Smith chart format. Use the marker function to move the diamond around the measurement trace to identify where different frequency points are located, e.g. edges of the pass band. These reflection parameters will exhibit a much more complex response in the Smith chart than other elements which have been examined so far. When filtering functions are introduced with several interacting poles, the magnitude and phase inter-relationships become significantly more complicated than the simple phase delays that have been examined thus far. For this reason, the S_{11} and S_{22} traces on the Smith chart no longer appear as simple circular arcs. A single S-parameter measurement at one frequency is inadequate to characterize the network. Instead the S-parameters must each be measured as a function of frequency over the range of interest.

FieldFox Parts List for Teaching Labs 1 through 6

Teaching labs 1 through 6 all require a FieldFox handheld analyzer and parts listed in the table below. Additionally, Lab 2 requires a Keysight MXG signal analyzer that is used to generate CW, AM and FM signals.

Note regarding the FieldFox handheld analyzer: These Labs were written for the FieldFox N9914A 6.5 GHz analyzer. However, the N9914A can be substituted with any of the following models: N9913A, N9915A, N9916A, or N9917A. The N9918A can also be used but additional adapters will be necessary because it has 3.5 mm test port connectors while the N9914A has Type-N.

Note regarding the MXG RF analog signal generator: Other MXG signal generators can be used but an A Series is simpler to use than a B Series, as the AM modulation scheme is different between the A and B Series. The Lab was written based upon the A Series.

Item no.	Description	Manufacturer	Mfr. part no.	Vendor	Vendor part no.	Qty
C1	SMA (m) to SMA (m) coaxial cable assembly, RG58C/U, 50 ohm, 18 in. long			L-Com	CCS58A-1.5	2
C2	Type-N (m) to BNC (f) coaxial adapter, 50 ohm			L-Com	AXA-NMBF	2
C3	Type-N (m) to SMA (f) coaxial adapter, 50 ohm			L-Com	AXA-NMSF	2
C4	SMA (f) to SMA (f) coaxial adapter, knurled middle			L-Com	BA23	2
C5	SMA (m) to SMA (m) coaxial adapter, Au			L-Com	BA22	2
C6	SMA (m) to SMA (m) to SMA (m) coaxial T adapter			L-Com	BA18	2
C7	SMA (m) terminator, 50 ohm			L-Com	BTS5M	2
C8	SMA (f) terminator, 50 ohm			L-Com	BTS5F	2
D1	Center-loaded telescoping whip antenna with BNC (m) connector, 19 in. long			Radio Shack	20-006	1
D2	800 MHz scanner antenna with BNC (m) connector			Radio Shack	20-283	1
D7	SMA coaxial attenuator, 10 dB, DC to 6 GHz, 50 ohm			Mini-Circuits	VAT-10+	1
D8	SMA coaxial band pass filter, 70 MHz, (63 to 77 MHz), 50 ohm			Mini-Circuits	SBP-70+	1
D9	SMA coaxial power splitter/combiner, 2-way, 90°, 50 ohm, 80 to 120 MHz			Mini-Circuits	ZMSCQ-2-120+	1
D10	SMA coaxial bi-directional coupler, 50 ohm, 50 W, 10 to 600 MHz			Mini-Circuits	ZFBDC20-62HP+	1
C9	SMA male shorting cap, Au	Amphenol Connex	132331_	Digi-Key	ACX2070-ND	1
A1	SMA (f) PCB jack, 50 ohm, 3 GHz, PTFE, Zn alloy/Au plated	TE Connectivity	5-1814832-1	Digi-Key	A97594-ND	1
A2	SMA (f) PCB jack, 50 ohm, 3 GHz, PTFE, brass/Ni plated	Linx Technol- ogies	CON SMA001	Digi-Key	CON SMA001-ND	1
A3	Thumbwheel trimpot, 500 ohm, 0.5 W, PC pin, cermet, single turn, top adjust	Bourns	3352T-1-510LF	Digi-Key	3352T-501LF-ND	1
A4	Resistor, 100 ohm, 1/4 W, 1%, axial lead metal film, 100 ppm/C	Yageo	MFR-25FBF52-100R	Digi-Key	100XBK-ND	1
A5	Capacitor, 22 pF, 100 V, 5%, radial lead ceramic disk, COG-NPO	Vishay	D220J20C0GH-63L6R	Digi-Key	1429PH-ND	1
A6	DIP8 machine pin socket, 0.100 pitch, 0.300 spacing, 30 µin Au plated	Mill-Max Mfg. Corp.	110-13-308-41-001000	Digi-Key	ED56083-ND	1
A7	Hook up wire, 22 AWG, solid, tinned Cu, black 300 V PVC insulated, 100 ft. spool	Alpha Wire	3051/1 BK005	Digi-Key	A3051B-100-ND	1

FieldFox Parts List for Teaching Labs 1 through 6 *continued*

Item no.	Test equipment	Manufacturer	Mfr. part no.	Vendor	Vendor part no.	Qty
T1	FieldFox handheld analyzer	Keysight	N9914A		N9914A	1
	– Option 233: Spectrum analyzer					1
	– Option 210: VNA transmission/reflection					1
	– Option 211: VNA full 2-port S-parameters					1
T2	MXG RF analog signal generator	Keysight	N5181A		N5181A	1
	– Option UNT: AM, FM, phase modulation					1

This application note was created by Professor Bruce Darling, from University of Washington's Electrical Engineering Department, in collaboration with Keysight Technologies' handheld team within the Component Test Division. The content is designed to complement an introductory course in undergraduate electromagnetics.

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